

Apparent superluminality and the generalized Hartman effect in double-barrier tunneling

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Recent papers suggest that tunneling wave packets traverse the region of allowed propagation between two potential barriers with superluminal group velocity and in a time independent of barrier separation. This phenomenon has been termed the “generalized Hartman effect” and extended to multiple barriers. Here we show that this delay time is not a transit time but a cavity lifetime. It does not imply superluminal velocity. Reported experimental verifications of this effect are reinterpreted.

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I. INTRODUCTION

The issue of tunneling times has been controversial for decades [1–8]. Part of the controversy stems from the fact that several tunneling time definitions predict effective tunneling velocities that can exceed the speed of light c in vacuum. These predictions apply to both the quantum mechanical tunneling of particles through potential barriers and the classical penetration of electromagnetic waves through evanescent regions. One well known prediction, termed the Hartman effect, states that the tunneling time, as measured by the delay of the peak of a wave packet, becomes independent of barrier length for reasonably opaque barriers [2,6–8]. This effect, considered one of the outstanding paradoxes of quantum mechanics [9], has been taken to imply superluminal and arbitrarily large group velocities for tunneling particles [4–9]. In recent papers we have shown that the Hartman effect can be explained by the saturation of stored energy (or particle number) with barrier length as a result of the exponential decay of the wave function with distance [10–15]. Since the group delay is proportional to the stored energy it saturates as the stored energy saturates. We pointed out that this delay is not a propagation delay but a *lifetime* and does not imply superluminal velocities. Since the wave packets are much longer than the barrier, what is observed is essentially a phase shift due to energy storage.

While the single-barrier problem is now better understood, there are lingering questions regarding tunneling through double barriers and multiple barriers in general. A recent paper by Olkhovsky, Recami, and Salesi, suggests that a particle or wave packet travels with superluminal, and indeed, infinite group velocity while crossing the region of allowed propagation separating two potential barriers [16]. This claim is based on the fact that in the opaque barrier limit used by these authors, the method of stationary phase yields a group delay which is independent, not only of the thickness of both barriers, but also of the separation L between the barriers. This phenomenon has been named the “generalized Hartman effect,” an apparent extension of the usual Hartman effect. The existence of this generalized Hartman effect has been accepted in the literature and several authors have published extensions of the theory [17–21] as well as experimental observations that appear to support this claim [22]. Esposito has found that a wave packet travels in

zero time across a multiple barrier [19] while Aharonov *et al.* find a similar phenomenon in tunneling through an array of δ function potentials [20,21]. These paradoxical predictions apply also to semiconductor superlattices, with the same surprising result that unbounded group velocities are permitted by such structures [23,24]. The experiment of Longhi *et al.* reports a group velocity as large as $5c$ and is purported to present “clear experimental evidence that, for opaque barriers, the traversal time is independent of barrier distance (*generalized* Hartman effect).” Indeed, an earlier experiment of Nimtz *et al.* done at microwave frequencies reported that the wave packet traversed the distance between barriers in zero time [25], a phenomenon apparently supported by numerical solutions of the Maxwell equations [26]. While the Hartman effect for forbidden regions can be understood fairly simply [10–15], a lack of dependence of group delay on distance in a region of allowed propagation is more troublesome since the wave functions in this region are oscillatory and do not decay. Attempts have been made to explain this surprising phenomenon on the basis of novel mechanisms such as “superoscillations” and “weak values” [20,21]. However, those explanations are not particularly transparent, at least to us.

In this paper we show explicitly that the calculated and measured group delays in double-barrier tunneling are cavity lifetimes that measure the decay of stored energy or the storage time of particles. They are not transit times in the sense of the time it takes a well defined point particle or wave packet to travel from A to B , passing through every point in between. We further show that the so-called generalized Hartman effect is simply due to the neglect of the fact that in the opaque limit nothing survives the voyage through the first barrier. In other words, since the transmission through the first barrier vanishes in the opaque limit, there is no amplitude in the second barrier or in the allowed region between the two. Since there is no energy beyond the first barrier there is nothing to be stored and so the storage time is zero. Since the amplitude of the transmitted wave goes to zero, the phase is meaningless. It should not be surprising that the group delay becomes independent of both the length of the interbarrier region and the width of the second barrier. In short, the generalized Hartman effect is an artifact. Under nonresonant conditions, a sequence of opaque barriers is just not a meaningful concept: if the first one is opaque, the second one might as well not be there. On the other hand, if any

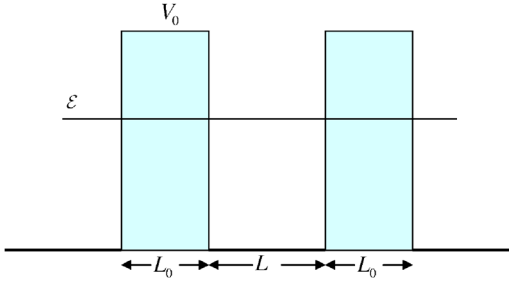


FIG. 1. (Color online) Geometry of the double barrier. The wave function is oscillatory in the allowed region and decays exponentially in the barrier regions.

energy makes it into the region of allowed propagation between the barriers, the storage time in that region should be proportional to the stored energy, which in turn is proportional to the length of that region. The delay then should be proportional to that length and not independent of it. It can be arbitrarily brief if little energy is stored.

II. THE DOUBLE BARRIER AS A FABRY-PÉROT CAVITY: GROUP DELAY AND DWELL TIME

The double-barrier geometry is shown in Fig. 1. The height of the barriers is V_0 , their width is L_0 , and their separation L . Outside the barriers the potential is zero. Incident from the left is a particle of mass m , wave number k , and energy $\mathcal{E} = \hbar^2 k^2 / 2m$. The stationary state problem of tunneling through two successive potential barriers in quantum mechanics is isomorphic to the transmission of electromagnetic waves through successive photonic band gap structures [22] or through successive waveguides below cutoff [25,26]. All these physical situations can be understood simply by viewing them as Fabry-Pérot cavities [27]. Consider therefore a Fabry-Pérot cavity (Fig. 2) with mirrors of reflectivity R and transmissivity T spaced by a distance L . For simplicity we assume the mirrors are lossless so that $R+T=1$. While the following discussion is couched in terms of electromagnetic waves, the results also hold for quantum wave packets. Inci-

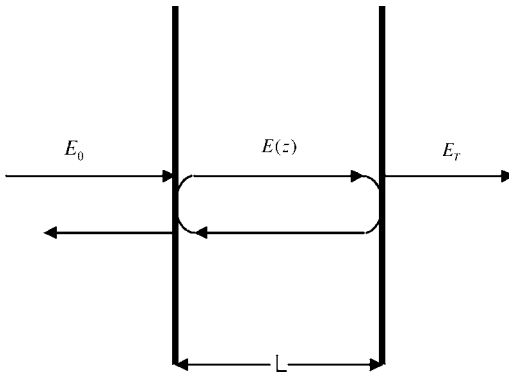


FIG. 2. Geometry of the Fabry-Pérot cavity. The transmitted field is related to the stored cavity field. The incident wave packet is assumed to be much longer than the cavity transit time so that interferences build up. This is the situation for all cases of distortionless tunneling.

dent from the left is a field of amplitude E_0 which gives rise to a cavity field $E(z)$ made up of the superposition of multiply reflected waves. If the cavity field is decomposed into a forward component E_F and a backward component E_B the boundary conditions at the input mirror require that

$$E_F(0) = \sqrt{T}E_0 + \sqrt{R}E_B(0), \quad (1a)$$

$$E_B(0) = \sqrt{R}E_F(0)e^{i2kL}, \quad (1b)$$

where the wavenumber $k = n\omega/c$, ω is the angular frequency of the wave, and n is the refractive index of the lossless medium mirrors within which the mirrors are embedded. Solving for $E_F(0)$ we find

$$E_F(0) = \sqrt{T}E_0 / (1 - Re^{i2kL}). \quad (2)$$

The total cavity field at any point is then

$$E(z) = \sqrt{T}E_0 (e^{ikz} + \sqrt{R}e^{i(2kL-kz)}) / (1 - Re^{i2kL}). \quad (3)$$

The field transmitted through the cavity is related to the forward part of the cavity field and is given by

$$E_T = TE_0 e^{ikL} / (1 - Re^{i2kL}). \quad (4)$$

The phase of the transmitted field is

$$\phi(k) = kL + \tan^{-1} [R \sin 2kL / (1 - R \cos 2kL)], \quad (5)$$

which yields the group delay (also known as the phase time or Wigner time) as [28]

$$\tau_g = \frac{d\tilde{\phi}}{d\omega} = \left[\frac{1 - R^2}{1 + R^2 - 2R \cos(2kL)} \right] \frac{L}{v}, \quad (6)$$

with $\tilde{\phi} = \phi(k) - kL$ and $v = c/n$. The group delay measures the time difference between the appearance of an incident wave packet peak at $z=0$ and a wave packet peak at $z=L$. These peaks are not necessarily related by a simple causal translation since the incident and transmitted pulses are not the same entity. In fact, the field at L at time t is related, not just to the field at $z=0$ at time $t-L/v$, but to the field at all prior times as a result of the multiple reflections in the cavity.

Another measure of the interaction of a wave packet with a barrier is the dwell time [29,30], defined as the ratio of the time-average stored energy to the incident power:

$$\tau_d = \langle U \rangle / P_{in}. \quad (7)$$

Here $\langle U \rangle = (1/2)\epsilon_0 n^2 A \int |E(z)|^2 dz$, $P_{in} = (1/2)\epsilon_0 n c |E_0|^2 A$, A is the cross sectional area of the cavity and ϵ_0 is the permittivity of free space. Using Eq. (3) we find that the time-average stored energy in the cavity is

$$\langle U \rangle = \frac{\epsilon_0 n^2 A L T [1 + R + 2\sqrt{R}(\sin 2kL)/2kL] |E_0|^2}{2(1 + R^2 - 2R \cos 2kL)}. \quad (8)$$

Upon dividing by the incident power we obtain the dwell time

$$\tau_d = \left(\frac{(1 - R^2) + 2T\sqrt{R}(\sin 2kL)/2kL}{1 + R^2 - 2R \cos(2kL)} \right) \frac{L}{\nu}. \quad (9)$$

The first term is identical to the group delay. The second term is a rapidly oscillating contribution to the stored energy which is very small for high frequency fields and gives rise to a self-interference delay [4,12]. It is negligible for cavities that are more than a few wavelengths long ($kL \gg 1$). Since the concept of group delay requires that a wave packet be at least several wavelengths long, we see that within its regime of applicability the group delay is identical to the dwell time. Even when this condition is not satisfied, group delay will be identical to dwell time at resonances and antiresonances of the cavity ($2kL = m\pi, m = 1, 2, \dots$).

The dwell time is the lifetime of stored energy escaping through both ends of the cavity [15]. For quantum particles it is the time spent in the barrier region, averaged over all incoming particles, regardless of whether a particle is reflected or transmitted [30]. It is not the time spent by any one particle. In a situation where most of the incoming particles are reflected, the dwell time will obviously be very short. To see how the dwell time and group delay relate to the escape rate through both transmission and reflection channels, first consider the case where the reflectivity of the mirrors is set to zero. The time-average stored energy in the transparent region of volume $V = LA$ is just $\langle U_0 \rangle = (1/2)\epsilon_0 n^2 |E_0|^2 AL$. The net energy flux transmitted in the forward direction through this lossless, reflectionless region is just the input power P_{in} . Upon dividing $\langle U_0 \rangle$ by P_{in} we get $L/\nu \equiv \tau_0$, the time it takes for all the energy stored in the region of length L to leave that region in the direction of the net flux and with velocity ν . Now allow the mirrors to have a finite reflectivity R . There will now be a backward reflected power P_r and a net transmitted forward power P_t such that, under quasistationary conditions, the two sum to yield the input power: $P_{in} = P_r + P_t$. The stored energy $\langle U \rangle$ in the volume between the mirrors will generally differ from its value in the absence of mirrors. Again we can find the time taken for all the stored energy to leave the cavity in the forward direction only by dividing that energy by the transmitted power: $\langle U \rangle / P_t = \tau_t$. The forward escape rate is thus $1/\tau_t$. Similarly, the time taken for all the energy to leave in the backward direction only is obtained by dividing the energy by the reflected power: $\langle U \rangle / P_r = \tau_r$. The backward escape rate is $1/\tau_r$. Since the energy is escaping simultaneously through both channels, the total escape rate is the sum $1/\tau = 1/\tau_r + 1/\tau_t$, or

$$\frac{1}{\tau} = \frac{P_r}{\langle U \rangle} + \frac{P_t}{\langle U \rangle}.$$

But this sum equals $P_{in}/\langle U \rangle$ which must therefore be the net rate of escape of energy from both ends of the cavity. This shows that the group delay or dwell time $\langle U \rangle / P_{in}$ is the lifetime of stored energy escaping through both ends. It is not necessarily a transit time in situations where the pulse is mostly reflected. Since it describes a simultaneous escape process from *both* channels, one cannot simply divide the length by the group delay and call it the group velocity for

forward transit. Unfortunately, this has been the standard practice [18].

The identity between group delay and cavity lifetime can also be seen from the usual definition of the Q of a cavity [31]:

$$Q = \omega \langle U \rangle / P_d, \quad (10)$$

which is the ratio of the stored energy to the power dissipated per cycle. The cavity lifetime is then defined as

$$\tau_c = Q / \omega = \langle U \rangle / P_d. \quad (11)$$

For a cavity without absorption the power dissipated is the power that escapes through the mirrors. By Poynting's theorem this power lost is equal to the input power P_{in} and hence we see that the group delay and dwell time are the same as the usual cavity lifetime.

The group delay is proportional to the time-average stored energy. The stored energy depends on the round trip phase shift $\theta = 2kL$ seen by the wave as it bounces back and forth within the cavity. Under resonant conditions, $\theta = 2m\pi$ ($m = 1, 2, \dots$), the recirculating fields add up in phase thereby enhancing the stored energy and increasing the storage time. The group delay or cavity lifetime at resonance is

$$\tau_g^m = \left(\frac{1 + R}{T} \right) \frac{L}{\nu}, \quad (12)$$

which can be made arbitrarily large compared to the transit time L/ν as the mirror transmission $T \rightarrow 0$. This resonant interaction forms the basis for "slow-light" devices and resonant-tunneling diodes [32]. On the other hand, when $\theta = (2m + 1)\pi$, antiresonant conditions obtain. Because of destructive interference between the recirculating phasors the stored energy is reduced below the value it would have had in the absence of the mirrors. Under these conditions the group delay is

$$\tau_g^{eff} = \left(\frac{T}{1 + R} \right) \frac{L}{\nu}. \quad (13)$$

This delay is always shorter than the cavity transit time and can be made arbitrarily small as $T \rightarrow 0$. Indeed, when $T = 0$ the group delay is zero. This of course does not mean that a pulse was transmitted through the Fabry-Pérot cavity in zero time. It simply means that no energy was stored between the mirrors, all of it was reflected, and no power was transmitted. We also see that the group delay is *not* independent of barrier separation (contrary to the findings in Refs. [16–18] but increases at a linear rate with that separation, in proportion to the stored energy. The rate of increase is proportional to the transmissivity T of the first mirror. It is definitely not a transit time but the cavity lifetime under nonresonant conditions. Figure 3 shows the normalized group delay τ_g/τ_0 , the normalized dwell time τ_d/τ_0 , and the normalized stored energy $\langle U \rangle / \langle U_0 \rangle$ as a function of round trip cavity phase shift. For $kL > 1$ all three quantities are identical. It is clear that the regions where the group delay is less than the free space transit time are exactly those regions where the stored energy is reduced below the free space value through destructive interference.

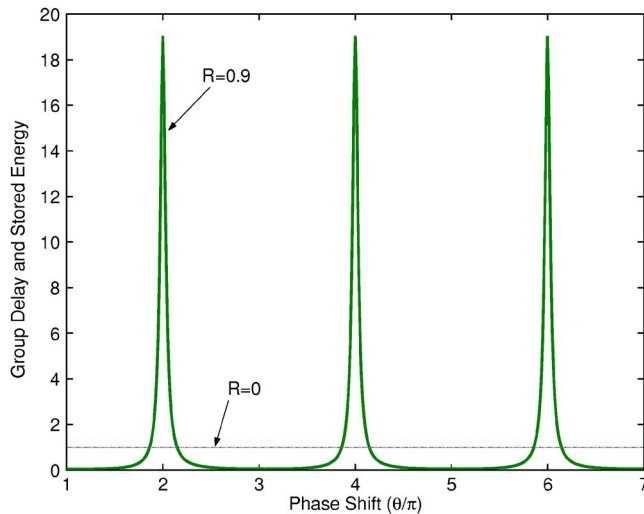


FIG. 3. (Color online) The normalized group delay (τ_g/τ_0), normalized dwell time (τ_d/τ_0), and normalized stored energy ($\langle U \rangle/\langle U_0 \rangle$) versus round trip phase shift in a Fabry-Pérot cavity with mirrors of reflectivity $R=0.9$. For $kL > \pi$, the three quantities are identical. The horizontal line at ordinate 1.0 shows the corresponding value for $R=0$.

As previously noted [11,14], tunneling without distortion requires narrowband pulses and hence pulses that are much longer than the cavity round trip time. Under these conditions when the pulse length is much greater than the cavity length, quasistationary conditions hold. The fact that the pulses have to be long compared to the structure is a very important consideration. It means that ultimately the duration of the tunneling process is just given by the length of the wave packet with an additional small delay due to energy storage. This delay is a small fraction of the pulse length, which means that the transmitted field is always beneath the envelope of the freely propagating pulse.

We can now explain the apparent superluminality seen in multiple-barrier tunneling. Typical pulsed experiments that report superluminal group “velocities” compare the arrival time t_1 of a peak for a reference pulse that has traversed a distance L in a barrier-free region with the arrival time t_2 of the peak of the transmitted fraction of a pulse that has interacted with a barrier [22,33,34]. For the reference pulse, the time t_1 is the time for all the energy $\langle U_0 \rangle$ stored in the region of length L to leave in the forward direction, given an input flux P_i : $t_1 = \langle U_0 \rangle / P_i$. For the tunneling pulse, given the same incident power P_i , the stored energy is much less than in the case of the reference pulse. Let that stored energy be $a\langle U_0 \rangle$ where $a \ll 1$. There are two channels of escape for the stored energy: the reflection (front) channel and the transmission (back) channel. The lifetime of this stored energy, escaping through both channels is $t_2 = a\langle U_0 \rangle / P_i$. Since $a \ll 1$ and P_i is the same, t_2 will be much smaller than t_1 by the factor a . This explains why the transmitted pulse peaks sooner than the reference pulse and why the delay time gets shorter as the transmission of the structure decreases. Every portion of the slowly varying input field suffers the same delay. This delay time is not the transit time for forward traverse but the non-resonant cavity lifetime of stored energy leaving through

both ends. (Note that since most of the power is reflected, we have $P_i \approx P_r$; hence we can also write $t_2 = a\langle U_0 \rangle / P_r$, which is the lifetime for stored energy leaving through the reflection channel.) To reiterate: the peak of the tunneled pulse occurs sooner because there is less stored energy for the same input power. Thus, what is really measured in “superluminal” tunneling experiments is just the off-resonant cavity lifetime.

III. DOUBLE-BARRIER EXPERIMENT RECONSIDERED

We now take a closer look at the double-barrier experiment involving cascaded fiber Bragg gratings [22]. This experiment has been taken to be a confirmation of the generalized Hartman effect: a lack of dependence of group delay on barrier separation. The gratings are of length L_0 each and are separated by a distance L . The strength of the gratings is characterized by a coupling constant κ . The power transmission of a grating of length L_0 at the Bragg frequency is

$$T = 1/\cosh^2(\kappa L_0), \quad (14)$$

while the overall transmission of the structure under anti-resonant conditions is given by

$$S = 1/\cosh^2(2\kappa L_0). \quad (15)$$

This is just the transmission of a grating of length $2L_0$. The group delay at midgap and away from Fabry-Pérot resonances can be written [22]

$$\tau_g = \tau_1 + \tau_2,$$

where

$$\tau_1 = \sqrt{1 - S}/\kappa\nu = \tanh 2\kappa L_0/\kappa\nu,$$

$$\tau_2 = \sqrt{SL}/\nu = [T/(1 + R)]L/\nu,$$

with $R = \tanh^2 \kappa L_0$. The first term τ_1 is independent of the separation between the two barriers and is equal to the group delay of a single barrier of length $2L_0$. [For quantum particles $\tau_1 = 2(\tanh 2\kappa L_0)/\kappa\nu$, where the factor of 2 comes from the self-interference delay in front of the barrier]. In the limit of an opaque barrier ($\kappa L_0 \gg 1$), $S \rightarrow 0$ and hence τ_1 saturates at the value of $1/\kappa\nu$, independent of barrier width. This is the usual Hartman effect which we have explained on the basis of saturation of the energy or number of particles stored in the barrier [10–15]. The second term τ_2 is proportional to the separation between the barriers and to the transmission of the barriers. Since $T \leq 1$, this term is less than the free propagation time L/ν of light across that distance. In fact, for an opaque barrier, where $T \rightarrow 0$, this contribution to the group delay vanishes entirely, approaching zero as $2e^{-2\kappa L_0}(L/\nu)$. This has been taken to mean that the light wave covers the distance L between the barriers in zero time and hence travels with infinite velocity [16–18]. We disagree with this interpretation. The group delays seen here are not necessarily propagation delays but are a measure of cavity lifetime. These delays are equal to the sum of dwell times in each of the regions and are proportional to the energy stored. The energy in the interbarrier region is proportional to the length

L of this region. However, it is also proportional to the transmission of the first barrier. If the transmission of the first barrier is zero then no light enters the interbarrier region and hence the “lifetime” there is zero. This is not a transit time. The lack of independence of the group delay on the barrier separation L for $T \rightarrow 0$ (the so-called generalized Hartman effect) is thus seen as an artifact resulting from the absence of light in the regions beyond the first barrier. On the other hand, under resonant conditions Eq. (12) shows that the lifetime will grow exponentially with the thickness of the barriers as $e^{2\kappa L_0(L/\nu)}$.

An examination of the experimental results confirms all our conclusions. We first note that the pulse length of 1.3 ns is much greater than the transit time of a light front through the medium (0.2 ns) and hence, as in all other reports of superluminal propagation, quasistatic conditions hold. Contrary to the descriptions based on “reshaping,” the transmitted pulse does not come from just the early tail of the incident pulse. It is the result of the cavity releasing energy stored from all earlier times. Second, the authors do in fact observe a dependence of “tunneling time” on barrier separation. Their plot of tunneling time versus barrier separation does show a rise, the slope of which should be given by T/ν . We can actually estimate the transmission from the slope of their measured delays. The interbarrier delay increases from a value of about 32 to 50 ps as the barrier separation is increased from 10 to 50 mm. This yields a slope of 4.5×10^{-10} s/m. Using the background refractive index of 1.452 we find a power transmission of about 0.9%. This is not far from the value of 0.8% given in Ref. [22] as the minimum transmission of the cascaded gratings. Since more than 99% of the incident energy is reflected, the measured delay cannot be associated with a forward traversal time.

The arguments presented here hold also for the case of multiple rectangular barriers and for arrays of δ function potentials. For example, in Ref. [19] it is found that the phase time for a system of N opaque barriers depends neither on the barrier width and interbarrier distance nor on the number of barriers. All these results can be explained by the vanishing of the transmission amplitude: if no energy or par-

ticles can be stored, the storage time is zero. It does not imply infinite velocity.

IV. CONCLUSIONS

We have shown explicitly that the group delay and dwell time in tunneling are storage times of energy and of particles and should not be associated with a forward transit group velocity in situations where a long wave packet (longer than the barrier) is mostly reflected. In particular, we have shown that for double-barrier tunneling, the lack of dependence of tunneling time on barrier separation (generalized Hartman effect) is an artifact of the absence of stored energy in the interbarrier region when the transmission of the first barrier goes to zero. The group delay in fact increases linearly with barrier separation when the transmission is nonzero.

The association of the transmission group delay with a transmission group velocity in highly reflective systems leads to serious logical difficulties. Such a connection requires that a pulse travels faster and faster as the transmission of the structure is reduced: the more repulsive the barrier the faster the wave packet wants to travel through it. This leads to the absurd conclusion that the pulse somehow travels with infinite velocity when the transmission of the structure has been reduced to zero. Such a logical conundrum is removed with the correct interpretation of the group delay in these cases as a lifetime and not a transit time. If the group velocity is to remain a meaningful physical concept as the speed with which a quantum particle travels, we must clearly disallow the possibility of such infinite values. Ultimately, experiments measure delay times and not velocities. Only if it can be shown that the delay is a transit time can one divide the path length L by the delay τ_g to obtain a meaningful velocity.

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- [1] L. A. MacColl, *Phys. Rev.* **40**, 621 (1932).
 - [2] T. E. Hartman, *J. Appl. Phys.* **33**, 3427 (1962).
 - [3] M. Buttiker and R. Landauer, *Phys. Rev. Lett.* **49**, 1739 (1982).
 - [4] E. H. Hauge and J. A. Støvneng, *Rev. Mod. Phys.* **61**, 917 (1989).
 - [5] Th. Martin and R. Landauer, *Phys. Rev. A* **45**, 2611 (1992).
 - [6] V. S. Olkhovsky and E. Recami, *Phys. Rep.* **214**, 340 (1992).
 - [7] R. Y. Chiao and A. M. Steinberg, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1997), Vol. 37, p. 345.
 - [8] *Time in Quantum Mechanics*, edited by J. G. Muga, R. Sala-Mayato, and I. L. Egusquiza (Springer, Berlin, 2002).
 - [9] *Mysteries, Puzzles, and Paradoxes in Quantum Mechanics*, edited by R. Bonifacio (AIP, Woodbury, NY, 1999).
 - [10] H. G. Winful, *Opt. Express* **10**, 1491 (2002).
 - [11] H. G. Winful, *Phys. Rev. Lett.* **90**, 023901 (2003).
 - [12] H. G. Winful, *Phys. Rev. Lett.* **91**, 260401 (2003).
 - [13] H. G. Winful, *Nature (London)* **422**, 638 (2003).
 - [14] H. G. Winful, *IEEE J. Sel. Top. Quantum Electron.* **9**, 17 (2003).
 - [15] H. G. Winful, *Phys. Rev. E* **68**, 016615 (2003).
 - [16] V. S. Olkhovsky, E. Recami, and G. Salesi, *Europhys. Lett.* **57**, 879 (2002).
 - [17] E. Recami, *J. Mod. Opt.* **51**, 913 (2004).
 - [18] V. S. Olkhovsky, E. Recami, and J. Jakiel, *Phys. Rep.* **398**, 133 (2004).
 - [19] S. Esposito, *Phys. Rev. E* **67**, 016609 (2003).
 - [20] Y. Aharonov, N. Erez, and B. Reznik, *Phys. Rev. A* **65**, 052124 (2002).
 - [21] Y. Aharonov, N. Erez, and B. Reznik, *J. Mod. Opt.* **50**, 1139

- (2003).
- [22] S. Longhi, P. Laporta, M. Belmonte, and E. Recami, Phys. Rev. E **65**, 046610 (2002).
- [23] P. Pereyra, Phys. Rev. Lett. **84**, 1772 (2000).
- [24] J. K. Tomfohr, O. F. Sankey, and S. Wang, Phys. Rev. B **66**, 235105 (2002).
- [25] G. Nimtz, A. Enders, and H. Spieker, J. Phys. I **4**, 565 (1994).
- [26] A. P. Barbero, H. E. Hernández-Figueroa, and E. Recami, Phys. Rev. E **62**, 8628 (2000).
- [27] H. Yuming, J. Phys. C **21**, L23 (1988).
- [28] J. Y. Lee, H-W. Lee, and J. W. Hahn, J. Opt. Soc. Am. B **17**, 401 (2000).
- [29] F. T. Smith, Phys. Rev. **118**, 349 (1960).
- [30] M. Buttiker, Phys. Rev. B **27**, 6178 (1983).
- [31] A. Yariv, *Optical Electronics in Modern Communications*, 5th ed. (Oxford University Press, New York, 1997), p. 144.
- [32] L. L. Chang, L. Esaki, and R. Tsu, Appl. Phys. Lett. **24**, 593 (1974).
- [33] A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, Phys. Rev. Lett. **71**, 708 (1993).
- [34] Ch. Spielmann, R. Szipocs, A. Stingl, and F. Krausz, Phys. Rev. Lett. **73**, 2308 (1994).